Class 9 Mathematics 2020 – 21

Chapter – 1 Notes Real Numbers

Videos watchin my youtube Naiki raju academy Playlist Real Numbers class 9

1. Real Numbers

1. REAL NUMBERS

- Number is a mathematical object used to count, measure and label.
- In Indian system, numbers are expressed by means of symbols 1, 2 ,3 ,4, 5, 6, 7, 8, 9 and 0 called digits.
- We can express a number in two ways.
- Representation of a number in figures is called **notation** and
- Expressing a number in words is called **numeration.**
- We use place value system to represent a number
- In Hindu-Arabic System we use ten symbols, namely 0, 1, 2 ,3 ,4, 5, 6, 7, 8, 9 we call them digits
- A number is denoted by a group of digits called **Numeral**
- Place value chart in Hindu Arabic system

Numbers are written by using the symbols 0,1,...9 with each symbol getting a value depending on the place it occupies

Face value of a digit in a numeral: It is the value of the digit itself.

Place value of or Local value of a digit in numeral.

It is the face value of the digit multiplying by the place value at which is situated.

Place value of the digit = face value of the digit × value of the place.

100 thousands for 1 lakh
1 million for 1 million
10 millions for a1 crore,
1 billion for a 1 arab or 100 crores.

We have Four fundamental Operation in numbers $+, -, \times, \div$

Subtraction: The operation of subtraction is an inverse process of addition:

Multiplication :- Multiplication is repeated addition.

Division:- Division is the inverse of multiplication

Division is repeated subtraction.

A number is divided by a non zero number b gives a quotient q and a remainder r then

$a = bq + r, where\ 0 \le r < b$

$$Dividend = (divisor \times quotient) + remainder$$

- ❖ Compare symbols in mathematics $<, > =$
- ❖ The division by zero is not defined or meaning less.

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Types of Numbers

Numbers can be classified into sets the same number can be written in many different ways.

1. Natural numbers (N),
2. whole Numbers (W),
3. integers (Z),
4. Rational Numbers (Q),
5. Irrational numbers ($Q^{|}$),
6. Real numbers (R),
7. Complex Numbers (C)

Natural numbers:- All Counting numbers are called Natural numbers, denoted by N

Natural numbers are {1, 2, 3 . . .}

The smallest natural number is 1 and the Natural numbers are infinite.

No largest natural number exists.

Whole Numbers:- All counting numbers together with zero are called whole numbers.

whole numbers denoted by W

whole numbers W= {0 , 1, 2, 3, . . .}

0 is a whole number which is not a natural number.

Successor of a whole number:-

If we add 1 to a given whole number , we get the next whole number, called its successor.

Predecessor of a whole number:

One less than a given whole number (other than 0) is called its predecessor.

Between's: If we consider any two consecutive whole numbers, clearly there is no whole number between them. But if we consider any two non consecutive whole numbers, clearly there is lie a whole number between them.

Integers : All counting numbers , 0 and negatives of counting numbers are called integers.

Negative integers are usually written a negative sign.

Integers are denoted by Z or i

$$integers = \boldsymbol{z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

$\{1, 2, 3 \ldots\}$ *is the set of positive integers*

$\{-1, -2, -3, -4, \ldots \ldots\}$ *is the set of negative integers*

0 *is neither positive nor negative.*

The set of all integers is represented by Z

❖ *Z comes from the* ***German word " Zahlen "*** *which means* ***'to count'***

Fractional Number :- A fraction means a part of a group or of a whole.

In the fraction the top number is called numerator,

where as the bottom number is called denominator.

Example : $\frac{4}{5}, \frac{8}{3}, \frac{9}{4}$ etc. are fractional numbers.

- If $p < q$, then it is a proper fraction and
- if $p \ge q$ then it is an improper fraction :

Rational Number

: – A number which can be expressed as $\frac{p}{q}$ where both p and q are integers and $q \neq 0$ is called **Rational Number**.

- When all the integers and fractions are put together ,then they are called rational number.
- Rational Number are denoted by Q (Q means quotient) ,

$$Q = \left\{\frac{p}{q},\ p, q \in Z,\ q \neq 0\right\}$$

- For example : $\frac{-5}{6}, \frac{8}{9}, \frac{-7}{11}$ etc. are rational numbers.
- All fractions are rational numbers but all rational numbers need not necessarily be fractions.

Property of density: –

- We can find infinite rational numbers between any two rational numbers. This property is called **Density Property**.

- Rational numbers can be either in the form of **terminating** decimals or **non-terminating repeating** decimals.
- Division by zero not defined
- $\frac{1}{a}$ is called multiplicative inverse of a , $a \neq 0$
- multiplicative inverse of 1 is 1

❖ Every rational number can be expressed either as a terminating decimal or as a non terminating recurring decimal.

- $\frac{2}{4} = 0.5$ the division is exact no remainder is left remainder is zero

  The quotients of such divisions are called terminating decimal

- $\frac{10}{3} = 3.333\ldots$

  the division never ends. No mater how long it continues the quotients of such divisions are called non terminating decimals.

- $\frac{4}{9} = 0.444\ldots = 0.\overline{4}$
- $\frac{11}{30} = 0.36666\ldots = 0.3\overline{6}$

  digit 3 is not repeated but digit 6 is repeated again and again.

- $\frac{4}{7} = 0.571428571428\ldots$

  The set of digits 571428 is repeated again and again.

- Rational numbers are of two kinds.
- 1) Terminating decimals, 2) Non terminating but repeating (recurring) decimals.

Terminating decimal: – A rational number which has no repeating digits in its decimal form is called terminating decimal.

- Exe: 3.25

Non – terminating repeating (recurring) decimals:- A rational number which has repeating digits in the decimal part, in its decimal form are called non-terminating repeating decimal.

- $Ex : 3.\overline{25}$

Period: The recurring part of the non-terminating but repeating decimal is called period.

- Exe: In $3.\overline{25}$ period is 25.

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Periodicity: The number of repeating digits in the decimal part of non-terminating but repeating decimal is called periodicity.

- Exe (1): In $3.\overline{25}$ periodicity is 2. Exe (2)): In $4.\overline{205}$ periodicity is 3.

Irrational Number :- The numbers which cannot be expressed in the form of $\frac{p}{q}$ form, where p, q are integers and $q \neq 0$ are called Irrational Numbers.
The decimal form of an Irrational Number is a non terminating or non recurring decimal
Square root of a number which is not perfect square, is an irrational number.
The square root of a number must be either a rational number or irrational number.

- Irrational numbers can be written as **non-terminating non-repeating** decimals.
- The irrational numbers can be classified into

i. **Surds** (Algebraic irrationals)

Transcendental numbers (Non – Algebraic)

Real Numbers: The combination of rational numbers and irrational numbers is known as the set of real numbers.
Real numbers are represented(denoted) by R
Every real number has a corresponding point on the number line and every point on the number line has a corresponding real number.
Hence, we call this line as the real number line.
Any point on the number line represents either rational number or irrational number.

Factor :- A number which divides the other number exactly is called a factor of that number.
A factor of number is an exact divisor of that number.
Which natural number is exactly divides the given number the natural number is factor of the given number
Factors of 4 = 1,2,4
Factors of 10 = 1, 2, 5, 10

- ❖ 1 is a factor of every number and is the smallest factor
- ❖ Every number is a factor of itself and is the largest factor

Multiples: – The products we get when a number is multiplied by the numbers 1, 2, 3,....are called the multiples of the given number.

- ❖ The numbers 5, 10, 15, 20, 25, ...are the multiple of 5.
- ❖ The numbers 2, 4, 6, 8, 10, 12.....are multiples of 2

Even Numbers: – All multiples of 2 are called even numbers.
Which number is divisible by 2 that is even number
Exam: 2, 4, 6, 8, 10, .etc. are all even numbers.

Odd Numbers: – Numbers which are not multiples of 2 are called odd numbers.
Which number is not divisible by 2 that is odd number
1, 3, 5, 7, 9, 11, 13, etc. are all odd numbers.

Prime numbers : The natural numbers which have only one and themselves as factors are called prime numbers (or simply primes)

- Numbers which can not be divided by any number other than 1 and number itself are known as prime numbers
- Which number has exactly two factors, namely 1 and itself, is called a prime number.
- 2, 3, 5, 7, 11, 13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97etc. are all prime numbers.

Composite Numbers :- Numbers having more than two factors are known as composite numbers.

- Numbers which can be divided by at least one of the number other than 1 and itself are known as composite numbers.

A number which has at least one more factor other than 1 and itself is called composite number.

- 1 is neither prime nor composite
- 2 is the lowest prime number
- 2 is the only even prime number, all other even number are composite numbers.
- Any prime number greater than 2 is an odd number.
- 4 is the smallest composite number.
- 9 is the smallest odd composite number

***Twin Primes* : –**

- If the difference between a pair of prime numbers is 2 they are called Twin primes.
- Pair of twin primes between 1 and 100 are

(3,5), (5,7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73),

***Prime triplet* :** A set of three consecutive prime numbers, differing by 2 is called a prime triplet.

- The only prime triplet is { 3, 5, 7}

***Perfect Number*:-** If the sum of all the factors of a number is two times the number then the number is called a perfect number.

- 6 is a perfect number, since the factor of 6 are 1, 2, 3, 6 and (1+2+3+6)= (2 × 6)
- 28 is a perfect number, since the factor of 28 are 1, 2, 4, 7, 14, 28 and $(1 + 2 + 4 + 7 + 14 + 28) = (2 \times 28)$

Co-primes: The numbers which have only "1" as the common factor are called **co-primes or relatively prime.**

If '1' is the only common factor of two numbers, then the two numbers are said to be relatively prime numbers.

Two numbers are said to be co-prime if they do not have a common factor other than 1

Any two prime numbers are always relatively prime to each other.

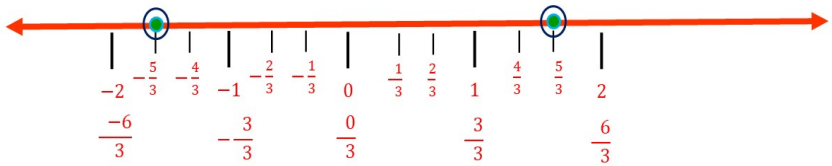
Square numbers :- A number can be written in the product of two equal factors that number is square Number 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

Cube numbers :- A number can be written in the product of three equal factors that number is square Number 1, 8, 27, 64, 125,216, 343, 512, 729, 1000

Class Notes

1. Represent $\frac{5}{3}$ and $-\frac{5}{3}$ on the number line

https://youtu.be/DLJ4lfVo8_c



Are the following statements True? Give reasons for your answers with an example

i. **Every rational is an intege**
ii. **Every integer is a rational number**
iii. **Zero is a rational number**

Solution :

i. False: For example ,$\frac{7}{8}$ is a rational number but not an integer.

ii. True: Because any integer can be expressed in the form $\frac{p}{q}, q \neq 0$ for example

$$-2 = \frac{-4}{2}$$

Thus it is a rational number.

$$(i.e.\ any\ integer\ 'b'\ can\ be\ represented\ as\ b = \frac{b}{1}$$

iii. True: Because 0 can be expressed as

$$0 = \frac{0}{2} = \frac{0}{3}$$

$$('0'\ can\ be\ represented\ as\ 0 = \frac{0}{x} where\ 'x'\ is\ an\ integer\ and\ x \neq 0)$$

Exmple 3: Find two rational numbbers between 3 and 4 by mean method

https://youtu.be/WCj8y7jh9Ys

The rational number lie between two rational number a and b $= \frac{a+b}{2}$

here a = 3 *b=4*

rational number between 3 *and* $4 = \frac{3+4}{2} = \frac{7}{2}$

rational number between 3 *and* $\frac{7}{2} = \frac{1}{2}\left(3 + \frac{7}{2}\right) = \frac{1}{2}\left(\frac{6+7}{2}\right) = \frac{13}{4}$

rational number between $\frac{7}{2}$ *and* $4 = \frac{1}{2}\left(\frac{7}{2} + 4\right) = \frac{1}{2}\left(\frac{7+8}{2}\right) = \frac{15}{4}$

Method 2: *Find two rational number between 3 and 4*

Write 3 *and* 4 *as rational numbers with denominator* 2+1= 3

$$\frac{3}{1} = \frac{3 \times 3}{1 \times 3} = \frac{9}{3}$$
$$\frac{4}{1} = \frac{4 \times 3}{1 \times 3} = \frac{12}{3}$$

Rational numbers between 3 and 4 $= \frac{10}{3}, \frac{11}{3}$

Method 2: *Find 5 rational number between 3 and 4*

Write 3 *and* 4 *as rational numbers with denominator* 5+1= 6

$$\frac{3}{1} = \frac{3 \times 6}{1 \times 6} = \frac{18}{6}$$
$$\frac{4}{1} = \frac{4 \times 6}{1 \times 6} = \frac{24}{6}$$

Rational numbers between 3 and 4 $= \frac{19}{6}, \frac{20}{6}, \frac{21}{6}, \frac{22}{6}, \frac{23}{6}$

Method 3: (decimal Numbers)

Find 5 rational number between 3 and 4

So many rational numbers between any two rational numbers

We know every rational can be write terminating decimal or non terminating recurring decimal

So the rational numbers between 3 and 4

3.1 , 3.26, 3.658, $3.\overline{4}$ 3.8745,

Ex 5: Express 3.28 in the form of $\frac{p}{q}$ (where p and q are integers $q \neq 0$)

https://youtu.be/xW4P1w0azy8

$$3.28 = \frac{328}{100} = \frac{2 \times 2 \times 82}{2 \times 2 \times 5 \times 5} = \frac{82}{25}$$

Ex 6: Express $1.\overline{62}$ in the form of $\frac{p}{q}$ (where p and q are integers $q \neq 0$)

https://youtu.be/u4CTYBxK0pM

$x = 1.\overline{62} = 1.626262 \ldots\ldots$ (1)

Limit = 2 so multiply eq (1) by 100

$100x = 100(1.626262 \ldots\ldots).$

$100x = 162.6262 \ldots\ldots$ (2)

Subtracting eq (2) from eq (1)

$100x = 162.6262 \ldots\ldots$

$-\ 1x = -\ 1.626262 \ldots\ldots$

$99x = 161.0000 \ldots\ldots$

$99x = 161$

$$x = \frac{161}{99}$$

$$\therefore 1.\overline{62} = \frac{161}{99}$$

Ex 6: Express $5.1\overline{45}$ in the form of $\frac{p}{q}$ (where p and q are integers $q \neq 0$)

https://youtu.be/u4CTYBxK0pM

$x = 5.1\overline{45} = 5.1454545 \ldots\ldots$ (1)

Limit = 2 so multiply eq (1) by 100

$100x = 100(5.1454545 \ldots\ldots).$

$100x = 514.54545 \ldots\ldots$ (2)

Subtracting eq (2) from eq (1)

$100x = 514.5454545 \ldots\ldots$

$-\ 1x = -\ 5.1454545 \ldots\ldots$

$99x = 509.40000 \ldots\ldots$

$99x = 509.4$

$$x = \frac{509.4}{99} = \frac{5094}{990}$$

$$\therefore 5.1\overline{45} = \frac{2 \times 3 \times 3 \times 283}{2 \times 3 \times 3 \times 55} = \frac{283}{55}$$

Find square root by the division method

1089; 67081 ; 100861849 ;

90.25 , 12544; 234.7024

https://youtu.be/zhiLjR4UNcc

Find the square root of $\sqrt{2}$ value upto 3 decimals .
square root 2 value upto 3 decimals .

https://youtu.be/u4UuFuZhKwY

Step 1 : *After 2, plac decimal point*

Step 2 : after decimal point write 0's

Step 3 : Group '0' in pairs and put a bar over them

Step 4 : Then after follow the method of find the square root of perfect square

$\sqrt{2} = 1.41421 \ldots = 1.414$

	1.41421
1	$2.\overline{00}\ \overline{00}\ \overline{00}$
1	1
24	100
4	96
281	400
1	281
2824	11900
4	11296
28282	60400
2	56564
282841	383600
	282841
	10075900

Find the square root of $\sqrt{3}$ value upto 3 decimals .
square root 3 value upto 3 decimals .

Step 1 : *After 3, plac decimal point*

Step 2 : after decimal point write 0's

Step 3 : Group '0' in pairs and put a bar over them

Step 4 : Then after follow the method of find the square root of perfect square

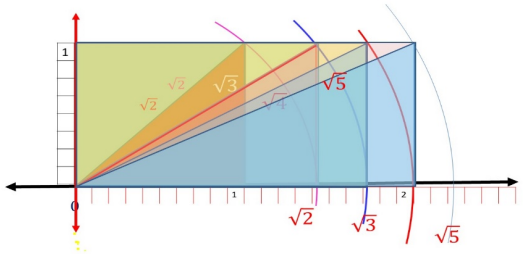
$\sqrt{3} = 1.73205 \ldots = 1.732$

	1.73205
1	$3.\overline{00}\ \overline{00}\ \overline{00}$
1	1
27	200
7	189
343	1100
3	1029
3462	7100
2	6924
34640	17600
0	00000
346405	1760000
	1732025
	27975

Represent $\sqrt{2}, \sqrt{3}, \sqrt{5}$ on number line

by power point presentation

by hand https://youtu.be/tUTngAfp4EM



Example 9 : – Find any two irrational numbers between $\frac{1}{5}$ and $\frac{2}{7}$

https://youtu.be/kOWQ5Id8zdo

$\frac{1}{5} = 0.2$

$\frac{2}{7} = 0.\overline{285714}$

0.201001000100001 ……

0.21212212321234 ……

If a and b two positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrrational numbr lying betwen a and b .

Example : −10 Find an irrational numbe between 3 and 4

an irrational number between 3 and 4 = $\sqrt{3 \times 4}$

$= \sqrt{3} \times \sqrt{4} = 2\sqrt{3}$

Example 11:- Examine whether the following numbers are rational or irrational

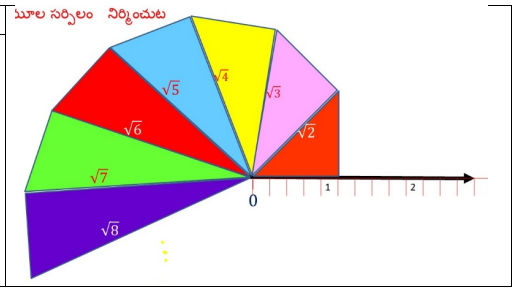
i) $(3+\sqrt{3})+(3-\sqrt{3})$ ***ii)*** $(3+\sqrt{3})(3-\sqrt{3})$ ***ii)*** $\frac{10}{2\sqrt{5}}$ ***iv)*** $(\sqrt{2}+2)^2$

1	$(3+\sqrt{3})+(3-\sqrt{3}) = 3+\sqrt{3}+3-\sqrt{3} = 6$ *Which is a rational number*
2	$(3+\sqrt{3})(3-\sqrt{3}) = 3^2-\sqrt{3}^2 = 9-3 = 6$ *Which is a rational number*
3	$\frac{10}{2\sqrt{5}} = \frac{2.5}{2\sqrt{5}} = \frac{2\sqrt{5}\sqrt{5}}{2\sqrt{5}} = \sqrt{5}$ *Which is an irrational number*
4	$(\sqrt{2}+2)^2 = (\sqrt{2})^2 + 2^2 + 2.2.\sqrt{2} = 2+4+4\sqrt{2} = 6+4\sqrt{2}$ *Which is an irrational number*

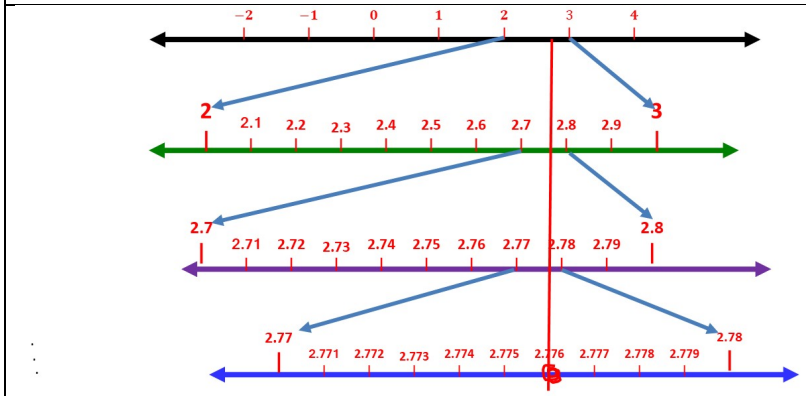
Constructing Square root spiral

https://youtu.be/VRUOACP6Sek

https://youtu.be/VaXv-foARqo



Visualise **2.776** *on the number line using successive magnification*

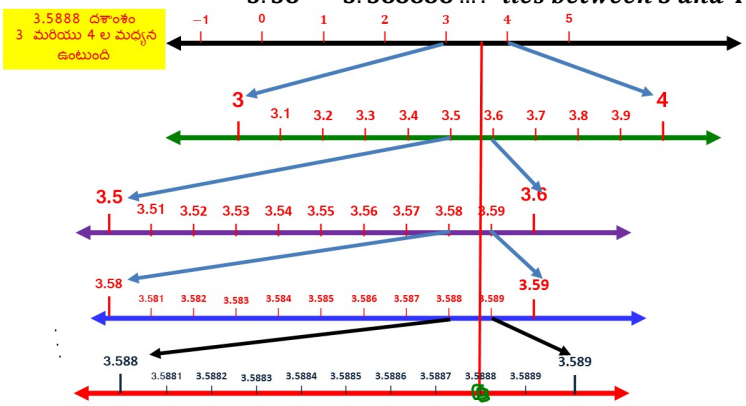


Ex12 :

Visualise **$3.5\overline{8}$** *on the number line, up to 4 decimal places using successive magnification*

https://youtu.be/kH2OP9sK4KI

$3.5\overline{8} = 3.588888\ldots.$ *lies between* 3 *and* 4



Surd

https://youtu.be/T7CQxrKpcQk

https://youtu.be/cENAKOcy_NY

Surd:- If **n** is a positive integer and a rational number a (>0) is not the **n^th** power of any rational number, then $\sqrt[n]{a}$ is called a **surd** of **n^th order**.

If a is a positive rational number which can not be expressed as the **n^th** power some rational number, then the rational umber $\sqrt[n]{a}$ or $(a)^{\frac{1}{n}}$ is called a surd or radical .

- The symbol $\sqrt[n]{\ }$ is called radical sign,
- **n** is called the order of the surd (or radical) and a is called radicand.

Type of surd

Quadratic surd: A surd of order 2 is called a "Quadratic surd".
Cubic Surd:- A surd of order 3 is called a "Cubic surd" .
Biquadratic surd:- A surd of order 4 is called a "Biquadratic surd"

Types of surds:-

Simple Surd (Monomial surd):- A surd which consists of a single term is called a simple or monomial surd
Ex: $\sqrt{7}, \sqrt[3]{5}, \sqrt[4]{9}$.....
Pure surds:- A surd which has unity only as rational factor, the other factor being irrational, is called a pure surd.
Ex:- $\sqrt{5}, \sqrt{29}, \sqrt{35}, \sqrt{55}$
Mixed Surd:- A surd which has a rational factor other than unity, the other factor being irrational is called a mixed surd.
If a is rational number and $\sqrt{b}$ is a surd then $a+\sqrt{b}, a-\sqrt{b}$ *and* $a\sqrt{b}$ are called mixed surds.
Ex: $2+\sqrt{3}, 5-\sqrt[3]{7}, 4\sqrt{5}$
Compound surd:A surd which is the sum or difference of two or more surds is called a compound surd.

An expression consisting of the sum or difference of two or more simple surds or the sum of or difference of a rational and a surd is called a compound surd.
Ex: $4+\sqrt{5}-\sqrt{8}$, $\sqrt{5}-\sqrt[4]{8}+7\sqrt{2}$
Binomial surds :-
The compound surd consisting of only two terms are called binomial surds.

Trinomial surds :-
The compound surd consisting of three terms are said to trinomial surds.
Polynomial surds:
The compound surd consisting of more than two terms are said to polynomial surds.
Like and unlike surds: (Similar surds and Dissimilar surds) :-
Two or more surds having the same irrational factor are known as similar or like surds", otherwise, they are called dissimilar unlike surds".
Laws of Radicals :-

$\sqrt[n]{a} = a^{\frac{1}{n}}$ $\quad \sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$ $\quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

$\sqrt[n]{(a)^p} = a^{\frac{p}{n}}$ $\quad \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = a^{\frac{1}{mn}}$

Comparison of two surds:-
Comparision of surds is possible only when they are of the same order.
If two surds are of the same order then the one whose radicand is larger is the larger of the two.
In order to compare the surds of different order and different base we first reduce them to the same order.

Operation of surds : –

Addition and Subtraction of like surds: Like surds can be added or subtracted by using the distributive laws.
Multiplication and division of surds: The surds of the same order can be multiplied or divided by using the rules.
If the given surds are not of the same order then they are converted to the lowest common order and then multiplied or divided as required

Rationalising factor (R.F) :-
If the product of two surds is a rational number, then each surd is called a rationalising factor of the other.
Note:-
- The R.F of a given surd is not unique
- If one R.F of a surd is known then the product of this factor by a non zero rational number is also a R.F. of the given surd.
- It is convenient to use the simplest of all R.F s of a given surd.

Rationalisation of surds: the process of converting a surd to a rational number by rationalising factor multiplying it with a suitable rationalising factor is called the rationalisation of the surd.
Monomial surd:-A surd consisting of only one term is known as a monomial surd.

Rationaling factor of monomial surd One of the rationlising factor of $(a)^{\frac{1}{n}}$ *is* $(a)^{\left(1-\frac{1}{n}\right)}$.

- Rationlising factor of $(a+\sqrt{b})$ is $(a-\sqrt{b})$
- Rationlising factor of $(a-\sqrt{b})$ is $(a+\sqrt{b})$
- Rationlising factor of $(\sqrt{a}+\sqrt{b})$ is $(\sqrt{a}-\sqrt{b})$
- Rationlising factor of $(\sqrt{a}-\sqrt{b})$ is $(\sqrt{a}+\sqrt{b})$

https://youtu.be/Ei8LxZa_DCY

write mixed surd $\sqrt{343}$

$$\sqrt{343}=\sqrt{7\times7\times7}=\sqrt{7^2}\times\sqrt{7}=7\sqrt{7}$$

write mixed surd $\sqrt[6]{729}$

$$\sqrt[6]{729}=\sqrt[6]{3\times3\times3\times3\times3\times3}=\sqrt[6]{3^6}=3$$

write mixed surd $\sqrt[3]{375}$

$$\sqrt[3]{375}=\sqrt[3]{3\times5\times5\times5}=\sqrt[3]{5^3}\times\sqrt[3]{3}=5\sqrt[3]{3}$$

write mixed surd $\frac{3}{4}\sqrt{8}$

$$\frac{3}{4}\sqrt{8}=\frac{3}{4}\sqrt{2\times2\times2}=\frac{3}{4}\sqrt{2^2}\times\sqrt{2}=\frac{3}{4}\times2\times\sqrt{2}=\frac{3}{2}\sqrt{2}$$

Home Work	Write Pure surd					
$5\sqrt{6}$	$2\sqrt[3]{4}$	$3\sqrt[4]{5}$		$\frac{3}{4}\sqrt{32}$	$10\sqrt{3}$	$\frac{2}{3}\sqrt{32}$
	Write Mixed Surd					
$\sqrt{80}$	$\sqrt[3]{72}$	$\sqrt[5]{288}$	$\sqrt[5]{320}$	$\sqrt{1350}$	$5\sqrt[3]{135}$	$\sqrt[3]{2025}$

$\sqrt[4]{\sqrt[3]{5}}=\sqrt[12]{5}$

Find rationalising factor $\sqrt{75}$

$$\sqrt{75}=\sqrt{25}\times3=\sqrt{5^2}.3=5\sqrt{3}$$

So $5\sqrt{3}\times\sqrt{3}=5\sqrt{9}=5\times3=15$

rational number

$\therefore$ ***rationalising factor of*** $\sqrt{3}$ ***is*** $\sqrt{75}$

Find rationalising factor $\sqrt[3]{36}$

sol :- $\sqrt[3]{36}=\sqrt[3]{2.2.3.3}=\sqrt[3]{2^2\,3^2}$

So $\sqrt[3]{2^2\,3^2}\times\sqrt[3]{2.3}=\sqrt[3]{2^3\times3^3}=\sqrt[3]{6^3}=6$

$\sqrt[3]{2.3}=\sqrt[3]{6}$

rationalising factor of $\sqrt[3]{36}$ *is* $\sqrt[3]{6}$

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Addition and subtraction of surds

Fundamental Operation on surds addition subtraction, multiplication, division Extra syllabus (order is different how to multiply and divide)	https://youtu.be/8vbkfMIlXw4

Distributive law $a\sqrt{c}+b\sqrt{c}=(a+b)\sqrt{c}$

Home Work

1. $5\sqrt{3}+10\sqrt{3}$	2. $2\sqrt{5}+\sqrt{125}$
3. $4\sqrt{7}-3\sqrt{252}+5\sqrt{343}$	4. $\sqrt{48}-\sqrt{72}-\sqrt{27}+2\sqrt{18}$
5. $\sqrt{125}-4\sqrt{6}+\sqrt{294}-2\sqrt{\frac{1}{6}}$	6. $\sqrt[3]{16}+\sqrt[3]{54}+\sqrt[3]{192}-\sqrt[3]{375}$
7. $5\sqrt{147}-\frac{4}{3}\sqrt{\frac{1}{3}}+7\sqrt{\frac{1}{3}}$	8. $\sqrt{294}-\sqrt{150}+2\sqrt{6}-3\sqrt{\frac{1}{6}}$
9. $5\sqrt{3}+2\sqrt{27}+\frac{1}{\sqrt{3}}$	10. $4\sqrt{2}-2\sqrt{8}+\frac{3}{\sqrt{2}}$
11. $\frac{1}{2}\sqrt{486}-\sqrt{\frac{27}{2}}$	12.

Multiplication and division of surds

The surds of the same order can be multiplied according to the law

$$\sqrt[n]{a}\times\sqrt[n]{b}=\sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}}$$

Find value of $\sqrt{14}\times\sqrt{21}$

Sol:- $\sqrt{14}\times\sqrt{21}$

$=\sqrt{14\times21}$

$=\sqrt{7\times2\times7\times3}$

$=\sqrt{7^2\times2\times3}=7\sqrt{6}$

Find value of $4\sqrt{28}\div3\sqrt{7}$

Sol:- $4\sqrt{28}\div3\sqrt{7}=\frac{4\sqrt{28}}{3\sqrt{7}}=\frac{4}{3}\sqrt{\frac{28}{7}}=\frac{4}{3}\sqrt{4}=\frac{4}{3}\times2=\frac{8}{3}$

Rationlising the Denominator **Rationalisation of monomial surds:**

E: find the simplest R.F of $\sqrt{75}$

$$\sqrt{75}=\sqrt{25}\times3=\sqrt{5^2}.3=5\sqrt{3}$$

So $5\sqrt{3}\times\sqrt{3}=5\sqrt{9}=5\times3=$

15 *rational number*

$\therefore$ $\sqrt{3}$ *is the simplest R.F. of* $\sqrt{75}$

find the simplest R.F of $\sqrt[3]{36}$

sol :- $\sqrt[3]{36}=\sqrt[3]{2.2.3.3}=\sqrt[3]{2^2\,3^2}$

So $\sqrt[3]{2^2\,3^2}\times\sqrt[3]{2.3}=\sqrt[3]{2^3\times3^3}=\sqrt[3]{6^3}=6$

$\sqrt[3]{2.3}=\sqrt[3]{6}$ *is the simpest R.F of* $\sqrt[3]{36}$

Rationalize the denominator in $\frac{2}{3\sqrt{3}}$

$$Sol: -\frac{2}{3\sqrt{3}}$$

The simplest R.F of $\sqrt{3}$ *is* $\sqrt{3}$ *itself*

$$\frac{2}{3\sqrt{3}} = \frac{2}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3\sqrt{9}} = \frac{2\sqrt{3}}{3\times 3} = \frac{2\sqrt{3}}{9}$$

Exercise -5 Home Work Write the simplest rationalising factor of

	$3\sqrt{3}$		$\sqrt{12}$		$\sqrt{108}$		$\sqrt[3]{192}$		

Express the following surds with rational denominators

	$\frac{3}{\sqrt{2}}$		$\frac{1}{3\sqrt{3}}$		$\frac{3}{\sqrt{15}}$		$\frac{2\sqrt{7}}{\sqrt{13}}$		$\frac{\sqrt[3]{7}}{\sqrt[3]{4}}$

Find the value up to three places of decimals of each of the following

It is given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$ $\sqrt{5} = 2.236$ $\sqrt{10} = 3.162$ (approximately)

	$\frac{1}{2\sqrt{2}}$		$\frac{1}{3\sqrt{3}}$		$\frac{3}{4\sqrt{5}}$		$\frac{2}{\sqrt{10}}$		

Rationalisation of binomial quadratic surds:-

Quadratic surds are the surds of second order. Two binomial expressions containing surds differing only in sign connecting them are said to be conjugate to each other.

Conjugate surds:-

If the sum and the product of two binomial surds is a rational number then they are called conjugate surds.

Two binomial surds which differ only in the signs (+/-) between them are called conjugate surds.

$2+\sqrt{7}, 2-\sqrt{7}$ *are conjugate surds.*

Conjugate of a surd of the form $a+\sqrt{b}$

If the product of two mixed surds $a+\sqrt{b}$ *and* $a-\sqrt{b}$ (where a and b are rational numbers) is a rational number, then $a+\sqrt{b}$ *and* $a-\sqrt{b}$ are called conjugate surds.

They are conjugate to each other.

$3+\sqrt{2}$ *is conjugate to* $3-\sqrt{2}$ *and vice versa.*

Write the conjugate of $5+\sqrt{2}$

Sol:- *conjugate of* $5+\sqrt{2}$ *is* $5-\sqrt{2}$

Since $(5+\sqrt{2})(5-\sqrt{2}) = 5^2 - \sqrt{2}^2 = 25-2 = 23$ a rational number

	Rationalise the denominator of $\frac{1}{\sqrt{2}}$
	https://youtu.be/d06tRjJX2-M
	Sol : $\frac{1}{\sqrt{2}}$ *here* $\sqrt{2}$ *is irrational number* *Rationlising factor of* $\sqrt{2}$ *is* $\sqrt{2}$ *multiplying the numerator and denominator of* $\frac{1}{\sqrt{2}}$ by $\sqrt{2}$ $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
	Rationalise the denominator of $\frac{1}{\sqrt{27}}$
	Sol : $\frac{1}{\sqrt{27}}$ *here* $\sqrt{27}$ *is irrational number* $\sqrt{27} = \sqrt{3\times 3\times 3} = \sqrt{3^2\times\sqrt{3}} = 3\sqrt{3}$ *Rationlising factor of* $3\sqrt{3}$ *is* $\sqrt{3}$ *multiplying the numerator and denominator of* $\frac{1}{\sqrt{27}}$ by $\sqrt{3}$ $\frac{1}{\sqrt{27}} = \frac{1}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3\times 3} = \frac{\sqrt{3}}{9}$
	Rationalise the denominator of $\frac{2}{3\sqrt{5}}$
	Sol : $\frac{2}{3\sqrt{5}}$ *here* $\sqrt{5}$ *is irrational number* *Rationlising factor of* $3\sqrt{5}$ is $\sqrt{5}$ *multiplying the numerator and denominator of* $\frac{2}{3\sqrt{5}}$ by $\sqrt{5}$ $\frac{2}{3\sqrt{5}} = \frac{2}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{3\times 5} = \frac{2\sqrt{5}}{15}$
Ex 18	**Rationalise the denominator of $\frac{1}{4+\sqrt{5}}$**
	Sol : $\frac{1}{4+\sqrt{5}}$ *here* $4+\sqrt{5}$ *is surd* *conjugate surd of* $4+\sqrt{5}$ is $4-\sqrt{5}$ *Multiplying the numerator and denominator of* $\frac{1}{4+\sqrt{5}}$ by $4-\sqrt{5}$ $\frac{1}{4+\sqrt{5}} = \frac{1}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}} = \frac{4-\sqrt{5}}{4^2-(\sqrt{5})^2}$ $= \frac{4-\sqrt{5}}{16-5} = \frac{4-\sqrt{5}}{11}$

Ex 19 — If $x = 7 + 4\sqrt{3}$ then find the value of $x + \frac{1}{x}$

https://youtu.be/B6IusZbux78

Sol: $\frac{1}{7+4\sqrt{3}}$

here $7+4\sqrt{3}$ is a surd

conjugate surd of $7+4\sqrt{3}$ is $7-4\sqrt{3}$

Multiplying the numerator and denominator of $\frac{1}{7+4\sqrt{3}}$ by $7-4\sqrt{3}$

$$\frac{1}{7+4\sqrt{3}} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{7-4\sqrt{3}}{7^2-(4\sqrt{3})^2}$$

$$= \frac{7-4\sqrt{3}}{49-(16\times 3)} = \frac{7-4\sqrt{3}}{49-48} = \frac{7-4\sqrt{3}}{1}$$

Ex : – 20 — **Simplify** $\frac{1}{7+4\sqrt{3}} + \frac{1}{2+\sqrt{5}}$

$$\frac{1}{7+4\sqrt{3}} + \frac{1}{2+\sqrt{5}}$$

conjugate surd of $7+4\sqrt{3}$ is $7-4\sqrt{3}$

conjugate surd of $2+\sqrt{5}$ is $2-\sqrt{5}$

$$\frac{1}{7+4\sqrt{3}} + \frac{1}{2+\sqrt{5}} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} + \frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$= \frac{7-4\sqrt{3}}{7^2-(4\sqrt{3})^2} + \frac{2-\sqrt{5}}{2^2-(\sqrt{5})^2}$$

$$= \frac{7-4\sqrt{3}}{49-(16\times 3)} + \frac{2-\sqrt{5}}{4-5}$$

$$= \frac{7-4\sqrt{3}}{49-48} + \frac{2-\sqrt{5}}{(-1)}$$

$$= \frac{7-4\sqrt{3}}{1} - \left(\frac{2-\sqrt{5}}{1}\right)$$

$$= 7-4\sqrt{3}-2+\sqrt{5} = 5-4\sqrt{3}+\sqrt{5}$$

Rationalise the denominaor $\frac{1}{3\sqrt{2}-2\sqrt{3}}$

$$\frac{1}{3\sqrt{2}-2\sqrt{3}}$$

conjugate surd of $3\sqrt{2}-2\sqrt{3}$ is $3\sqrt{2}+2\sqrt{3}$

$$\frac{1}{3\sqrt{2}-2\sqrt{3}} = \frac{1}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$$

$$= \frac{3\sqrt{2}+2\sqrt{3}}{(3\sqrt{2})^2-(2\sqrt{3})^2}$$

$$= \frac{3\sqrt{2}+2\sqrt{3}}{9\times 2-4\times 3}$$

$$= \frac{3\sqrt{2}+2\sqrt{3}}{18-12}$$

$$= \frac{3\sqrt{2}+2\sqrt{3}}{6}$$

Exponential form : –

$a \in R$, $a \times a \times a \times a \times a \ldots \times a$ (n times) write a^n

Identities of Exponents

1. $a^0 = 1\ (a \in R,\ a \neq 0)$
2. $a^m a^n = a^{m+n}\ (m, n \in Z)$
3. $\frac{a^m}{a^n} = \begin{cases} 1 & m = n \\ a^{m-n} & m > n,\ (m, n \in Z) \\ \frac{1}{a^{n-m}} & m < n\ (m, n \in Z) \end{cases}$
4. $(a^m)^n = a^{mn}\ (a \in R)$
5. $(ab)^m = a^m b^m\ (m, n \in Z)$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\ (n \in Z, b \neq 0)$
7. $a^{-n} = \frac{1}{a^n}\ (n \in N, a \in R\ a \neq 0)$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}\ (n \in w\ a \neq 0)$

Ex 21:		Simplify
	$(2)^{\frac{2}{3}}.(2)^{\frac{1}{3}}$	$(2)^{\frac{2}{3}}.(2)^{\frac{1}{3}} = (2)^{\frac{2}{3}+\frac{1}{3}} = (2)^{\frac{3}{3}} = 2^1 = 2$
	$\left(5^{\frac{1}{7}}\right)^4$	$(5)^{\frac{1}{7}\times 4} = 5^{\frac{4}{7}}$
	$\frac{3^{\frac{1}{5}}}{3^{\frac{1}{3}}}$	$\frac{3^{\frac{1}{5}}}{3^{\frac{1}{3}}} = 3^{\frac{1}{5}-\frac{1}{3}} = 3^{\frac{3-5}{15}} = 3^{\frac{-2}{15}}$
	$7^{\frac{1}{17}}.11^{\frac{1}{17}}$	$(7\times 11)^{\frac{1}{17}} = (77)^{\frac{1}{17}}$

If a and b are rational numbers, Find the values of and b $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$

https://youtu.be/_AU96i4-daA

Sol :– $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$

Rationalise factor $7+4\sqrt{3}$ is $7-4\sqrt{3}$

Multiplying the numerator and denominator of $\frac{1}{7+4\sqrt{3}}$ by $7-4\sqrt{3}$

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=\frac{5+2\sqrt{3}}{7+4\sqrt{3}}\times\frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$=\frac{35-20\sqrt{3}+14\sqrt{3}-8\sqrt{9}}{7^2-\left(4\sqrt{3}\right)^2}$$

$$=\frac{35-6\sqrt{3}-8\times3}{49-16\times3}$$

$$=\frac{35-6\sqrt{3}-24}{49-48}$$

$$=\frac{11-6\sqrt{3}}{1}$$

$$=11-6\sqrt{3}=a+b\sqrt{3}$$

$$a=11,\ b=-6$$

$\sqrt{10}=3.162$ (approximately) Find the value of three places of decimals of $\frac{1}{\sqrt{10}}$

$$\frac{1}{\sqrt{10}}=\frac{1}{\sqrt{10}}\times\frac{\sqrt{10}}{\sqrt{10}}=\frac{\sqrt{10}}{\sqrt{100}}=\frac{\sqrt{10}}{10}=\frac{3.162}{10}=0.3162$$

$$=0.316 \text{ correct to three dical places}$$

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simplify $4\sqrt{12}-\sqrt{50}-7\sqrt{48}$

Sol:- $4\sqrt{12}-\sqrt{50}-7\sqrt{48}$

$4\sqrt{4\times3}-\sqrt{25\times2}-7\sqrt{16\times3}$

$4\sqrt{2^2\times3}-\sqrt{5^2\times2}-7\sqrt{4^2\times3}$

$4\times2\times\sqrt{3}-5\times\sqrt{2}-7\times4\times\sqrt{3}$

$8\sqrt{3}-5\sqrt{2}-28\sqrt{3}$

$-20\sqrt{3}-5\sqrt{2}$

Simplify $4\sqrt{3}-3\sqrt{12}+2\sqrt{75}$

Sol:- $4\sqrt{3}-3\sqrt{12}+2\sqrt{75}$

$4\sqrt{3}-3\sqrt{4\times3}+2\sqrt{25\times3}$

$4\sqrt{3}-3\times2\times\sqrt{3}+2\times5\times\sqrt{3}$

$4\sqrt{3}-6\sqrt{3}+10\sqrt{3}$

$(4-6+10)\sqrt{3}$

$=(14-6)\sqrt{3}$

$=8\sqrt{3}$

Simplify $\sqrt[4]{81}-8\sqrt[3]{216}+15\sqrt[5]{32}+\sqrt{225}$